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Absence of a VVDZ discontinuity in AdS_{AdS}

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ABSTRACT: We clarify the role of gauge invariance for the theory of an AdS_4-brane embedded in AdS_5. The presence of a non-vanishing mass parameter even for the lightest KK mode of the graviton indicates that all of the spin-2 modes propagate five polarization states. Despite this fact, it was shown earlier that the classical theory has a smooth limit as the mass parameter is taken to zero. We argue that locality in the fifth dimension ensures that this property survives at the quantum level.

KEYWORDS: Extra Large Dimensions, AdS/CFT Correspondance, Classical Theories of Gravity.

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1. Introduction

Recently it was observed that a four-dimensional AdS theory derived from a brane embedded in AdS_5 reproduces correctly a four-dimensional theory of gravity, despite the presence of a mass parameter [1]. It was shown by M. Porrati [2] and I. Kogan et al. [3] that this is not in contradiction with the Van Dam Veltman Zakharov (VVDZ) [4] discontinuity, which in flat space says that the limit of a massive graviton as the mass goes to zero is not a massless graviton, because additional polarization states survive. However, it was observed in [5] that the VVDZ discontinuity could reappear for a single massive spin 2 excitation at the one-loop level.

In this paper we want to clarify the counting of degrees of freedom and the role of gauge invariance in the AdS_{AdS} setup.[1] In addition, we will argue on the basis of locality of the higher-dimensional theory that there cannot exist a VVDZ discontinuity, even on the quantum level.

Our outline is as follows. In the next section, we summarize the counting of degrees of freedom. In section 3, we discuss the fluctuation spectrum around an AdS_4-brane to show explicitly that the only propagating modes of the graviton are the transverse traceless modes. We speculate about the possibility of a non-local 4D effective description in section 4. In section 5 we will discuss possible quantum effects and will argue on the basis of locality in the fifth dimension, that no VVDZ discontinuity can appear even at the quantum level.

[1]We denote the theory of [1] in which an AdS_4-brane is embedded inside $_{AdS_5}$ as AdS_{AdS}.

2. Counting propagating degrees of freedom

The theory we consider is fundamentally a five-dimensional theory of gravity. Therefore, there are naively fifteen polarizations. It is clear that one can go into axial gauge, where there are only ten polarization states. We can now proceed in several different ways to further eliminate gauge artifacts. We can first choose Transverse-Traceless (TT) gauge, in which both the longitudinal states and the trace are eliminated for all the KK modes. This explicitly demonstrates the absence of a physical ghost scalar. After making this gauge choice, there are still three residual gauge invariances. However these gauge transformations do not fall off at the boundary of AdS. This means we can not eliminate normalizable fluctuations with these non-normalizable gauge transformations. If we expand the graviton into normalizable modes and a non-normalizable vector field as in [6], only the non-normalizable Stückelberg like vector field can be set to zero. We therefore remain with a tower of normalizable massive spin 2 modes, each with 5 degrees of freedom. So just from counting degrees of freedom, for our system the absence of the VVDZ discontinuity still looks puzzling. On the classical level the results of [2, 9] show that despite the mismatch in polarization states there is no discontinuity in the propagator. Without having gauge invariance to actually remove three polarizations, one would generically expect that the discontinuity would reappear when considering loop corrections. In section 4 we will argue that it is really locality in the fifth dimension which guarantees absence of the VVDZ discontinuity even at the quantum level.

3. Mode analysis for the AdS_4-brane

In this section, we explicitly analyze the modes of the AdS_4-brane, considered in ref. [1, 7, 8, 9] and find that there is no ghost in the spectrum; that is, the TT modes suffice and any apparent ghost is a gauge artifact [10]. We are using the same conventions and background solution as in [1]. The gauge transformations read

$$\begin{aligned} &h_{55} \longrightarrow h_{55} - 2e^{-2A}\xi^{5'}\,, \qquad h_{\mu\nu} \longrightarrow h_{\mu\nu} + (\nabla_\mu \xi_\nu + \nabla_\mu \xi_\nu) + 2A' g_{\mu\nu}\xi^5\,, \\ &h_{\mu 5} \longrightarrow h_{\mu 5} - e^{-2A}\nabla_\mu \xi^5 + g_{\mu\nu}\xi^{\nu'}\,. \end{aligned} \tag{3.1}$$

It is clear there is a choice of gauge where $h_{\mu 5} = h_{55} = 0$. We want to study transformations that leave this gauge choice invariant. They are required to satisfy

$$\xi_5 = \epsilon^5(x)\,, \qquad \xi_\mu = G\nabla_\mu \epsilon^5 + \epsilon_\mu(x) \tag{3.2}$$

with $G = \int_0^r e^{-2A(\tilde{r})} d\tilde{r}$.

The equations of motion demand

$$\begin{aligned} &55 :\ \left(e^{2A}h'\right)' = 0\,, \qquad 5\mu :\ -\frac{1}{2}\nabla_\mu h' + \frac{1}{2}\nabla^\rho h'_{\mu\rho} = 0\,, \\ &\mu\nu :\ e^{2A}\left(\frac{1}{2}h''_{\mu\nu} + 2A'h'_{\mu\nu}\right) - \frac{1}{2}\nabla^2 h_{\mu\nu} + \frac{1}{2}g_{\mu\nu}e^{2A}A'h' - \frac{1}{2}\nabla_\mu\nabla_\nu h + \\ &\qquad + \frac{1}{2}\left(\nabla^\rho\nabla_\mu h_{\nu\rho} + \nabla^\rho\nabla_\nu h_{\mu\rho}\right) + 3\Lambda h_{\mu\nu} = 0\,. \end{aligned} \tag{3.3}$$

with the boundary condition $h'_{\mu\nu} = 0$.

A generic solution to the bulk equations of motion can be brought into the form [1]

$$h_{\mu\nu} = h^{TT}_{\mu\nu} + 2G\,(\nabla_\mu \nabla_\nu - \Lambda g_{\mu\nu})\Phi - \frac{2}{3}\lambda g_{\mu\nu}\Phi \tag{3.4}$$

with Φ satisfying $\nabla^2\Phi = 4\Lambda\Phi$, where we have used the ϵ_μ degrees of freedom and the equations of motion to fix $G(0) = 0$. Both $h = -\frac{8}{3}\lambda\Phi$ and $\nabla_\mu h^{\mu\nu} = -\frac{2}{3}\lambda\nabla^\nu\Phi$ are r-independent. Notice that Φ is essentially the brane-bending [11] degree of freedom. This can be seen from the boundary conditions which are

$$h'_{\mu\nu} = 0 = h^{TT'}_{\mu\nu} + 2e^{-2A}(\nabla_\mu \nabla_\nu - \Lambda g_{\mu\nu})\Phi\,, \tag{3.5}$$

where TT stands for transverse traceless. In the absence of matter, we can use the ϵ_5 degree of freedom to eliminate Φ.

Although we introduced the Φ field in the previous analysis to follow refs. [1, 12, 13], we could have used gauge invariance from the start to see that the introduction of Φ was superfluous. A faster way to arrive at the same conclusion (that is tracelessness of the graviton) is to first gauge fix the generic solution to the $\mu 5$ and 55 as much as possible before studying the $\mu\nu$ equation. From the 55 equation, we get $e^{2A}h' = c(x)$. The boundary conditions $e^{2A(0)}h' = 0$ require $c(x) = 0$. So $h = f(x)$. As in a purely four-dimensional analysis, we then use the ϵ_μ gauge transformations to eliminate f. Similarly, the $\mu 5$ equations require $\nabla^\mu h_{\mu\nu} = g_\nu(x)$ and we can eliminate the longitudinal parts as well. The remaining $h_{\mu\nu}$ is now manifestly transverse and traceless. In both cases, it is crucial that there is only a single brane; a second boundary condition would ruin our ability to restrict to TT modes only. For example, our analysis would not apply to the GRS model [14].

In the presence of sources, one could follow the procedure outlined in [16]. There, one first finds the profile for the trace and longitudinal components of the metric. These no longer vanish but are given in terms of differential equations involving r only; that is, they do not propagate in the four-dimensional spacetime. One then uses these components as extra sources to propagate the massive TT modes. The upshot is that the only propagating degrees of freedom are the massive TT modes. There is no ghost in the spectrum.

4. Effective four-dimensional description

An interesting question is if AdS_{AdS} can be described by an effective 4d action. Since the mass of the lowest mode is of order Λ^2, it might be expected that the mass term arises from terms that are quadratic in the curvature. However it is easy to show that in four dimensions, local curvature squared terms change neither the mass for the zero mode, nor the AdS curvature. That is, the background solution is unchanged, and there is no mass generated for the linear perturbations. To see the latter, observe that to linear order in the fluctuation, and to second order in derivatives, the only gauge invariant combination is the one set to zero by the linearized Einstein equation, namely,

$$R^{(1)}_{MN} - \frac{1}{2}g_{MN}R^{(1)} + 3\Lambda h_{MN}\,.$$

Here, $R^{(1)}_{MN}$ and $R^{(1)}$ are the linearized Ricci tensor and Ricci scalar, respectively, while h_{MN} is the metric fluctuation. Thus, all linearized higher curvature terms are either proportional to this combination, or are traces and derivatives of it. To find the possible mass terms to first order in L^2, it is sufficient to substitute the solution to the zeroth order Einstein equation into the above equation of motion. As all higher curvature terms contain the above combination (set to zero by Einstein's equations), one finds that such terms vanish, and hence do not contribute a mass. This argument shows that one cannot construct a local effective lagrangian of metric fluctuations alone that reproduces the physics of the brane fluctuations.

Of course there are several other possibilities. One way out would be to have a non-local 4d effective action. It seems reasonable that this way one can produce a mass term of order Λ^2 from higher curvature terms by a mechanism analogous to the Debye mass in crystals. This non-local effective action might be the result of integrating out a CFT. In fact, it is clear that the construction of a 4D effective theory is subtle, since it is not even true that all regions of space see four-dimensional gravity.

5. Absence of the VVDZ discontinuity

As we emphasized in section 2, the massive 4d graviton always propagates 5 degrees of freedom. The absence of the VVDZ discontinuity in AdS_4 was established in [2, 9] by a detailed study of the propagator. Despite the difference in polarization states, the massive propagator in AdS has a smooth limit as the mass goes to zero. In the double scaling limit, sending both the cosmological constant and the mass to zero, modes whose m^2 go to zero faster than Λ asymptote to the massless graviton in 4d, while modes whose mass vanishes slower asymptote to a massive graviton in 4d, which has a different propagator from the massless graviton already at tree level, even when the mass parameter is taken to zero. It was questioned in [5], whether this property survives in the quantum theory.

In AdS_{AdS} the mass squared of the almost zero mode goes to zero as Λ^2, so that it can smoothly go over into the massless graviton, while the excited modes become the massive KK modes in the flat space limit. From a 4d point of view, both the cosmological constant and the mass will undergo renormalization. It is to be understood that a large AdS_4 requires a fine tuning of the quantum corrected cosmological constant. The question now really becomes whether the mass of the almost zero mode after quantum corrections still goes like Λ^2 and hence effectively reproduces massless physics, or whether it looks more like one of the KK-modes, with m^2 of order Λ, which would signal a quantum VVDZ discontinuity. From the 4d perspective, nothing seems to protect the mass of the almost zero mode.

However from the 5d point of view, locality guarantees that the low energy effective action is still of the same form as the tree level action, with effective tension, Newton's constant and bulk cc plus some higher derivative corrections. The one fine tuning which we know we have to do amounts to balancing effective tension versus effective bulk cc. Once we have done this, the mass squared of the almost zero mode will still go like Λ^2_{eff}, since once we use the effective action, the full quantum answer is given by the tree level analysis, using the effective parameters instead of the bare parameters is appropriated. Therefore there is no VVDZ discontinuity even at the quantum level in AdS_{AdS}.

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